

QUIZ 9 SOLUTIONS: LESSON 10
FEBRUARY 8, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

A 500 gallon tank initially contains 200 gallons of brine containing 25 pounds of dissolved salt. Brine containing 2 pounds of salt per gallon flow into the tank at a rate of 4 gallons per minute, and the well-stirred mixture flows out of the tank at a rate of 1 gallon per minute. Let $A(t)$ be the number of pounds of salt in the tank at t minutes.

$$\frac{dA}{dt} = [\text{Rate of salt in}] - [\text{Rate of salt out}]$$

1. [1 pt] Find the rate of the salt entering the tank in pounds per minute.

$$\left(\frac{2 \text{ lbs}}{1 \text{ gal}} \right) \left(\frac{4 \text{ gal}}{1 \text{ min}} \right) = \boxed{8 \text{ lbs/min}}$$

2. [2 pts] Find the rate of the salt leaving the tank in pounds per minute.

Well-stirred = average of salt of brine in gal in tank

gal in tank at time t :
 $200 + 4t - 1t$
 ↑ gal in ↑ gal out

$$\left(\frac{A \text{ lbs}}{200 + 3t \text{ gal}} \right) \left(\frac{1 \text{ gal}}{1 \text{ min}} \right) = \boxed{\frac{A}{200 + 3t} \text{ lbs/min}}$$

3. [5 pt] Find the general solution to $\frac{dA}{dt}$.

$$\frac{dA}{dt} = 8 - \frac{A}{200 + 3t}$$

$$\Rightarrow \frac{dA}{dt} + \frac{A}{200 + 3t} = 8$$

This is a FOLDE

Step 1: Find P, Q

$$P = \frac{1}{200 + 3t}, \quad Q = 8$$

Step 2: Find int factor

$$\int P(t) dt = \int \frac{1}{200 + 3t} dt$$

Integration by substitution

$$= \frac{1}{3} \ln(200 + 3t) = \ln(200 + 3t)^{1/3}$$

$$u = e^{\int P(t) dt} = e^{\ln(200 + 3t)^{1/3}} = (200 + 3t)^{1/3}$$

Step 3: Set up soln.

$$A \cdot (200 + 3t)^{1/3} = \int 8 \cdot (200 + 3t)^{1/3} dt$$

$$= 2(200 + 3t)^{4/3} + C \quad \leftarrow \text{Integration by Sub}$$

$$\Rightarrow \boxed{A = 2(200 + 3t) + C(200 + 3t)^{-1/3}}$$

$$w = 200 + 3t \quad \int \frac{1}{200 + 3t} dt = \int \frac{1}{3w} dw = \frac{1}{3} \ln w = \frac{1}{3} \ln(200 + 3t)$$

$$dw = 3 dt \Rightarrow dt = \frac{dw}{3}$$

$$\int 8(200 + 3t)^{1/3} dt = \int \frac{8}{3} w^{1/3} dw = \frac{8}{3} \left(\frac{3}{4} \right) w^{4/3} + C$$

$$= 2w^{4/3} + C = 2(200 + 3t)^{4/3} + C$$

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4. [1 pt] Find the particular solution to $\frac{dA}{dt}$.

$A(0) = 25$ means

$$25 = 2(200 + 3(0)) + C(200 + 3(0))^{-1/3} = 2(200) + C(200)^{-1/3}$$

$$-375 = C(200)^{-1/3} \Rightarrow C = -375(200)^{1/3}$$

$$A = 2(200 + 3t) - 375(200)^{1/3}(200 + 3t)^{-1/3}$$

5. [1 pt] How much salt is in the tank when the tank is full? Round your answer to 2 decimal places.

The tank is full when the total number of gal in the tank equals 500, so when $200 + 3t = 500$. Hence, $t = 100$ when the tank is full.

$$A(100) = 2(200 + 3(100)) - 375(200)^{1/3}(200 + 3(100))^{-1/3} \approx \boxed{723.7 \text{ lbs}}$$